Point-like sources of Euclidean non-abelian field in the instanton liquid

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An appraisal of average energy of Euclidean non-abelian point-like source in the instanton liquid is calculated. It behaves linearly increasing at large distances while unscreened. For the dipole in colour singlet state this energy increases linearly again with the separation increasing and its "tension" coefficient develops the magnitude pretty similar to the lattice approach and other model estimates. The situation of sources arbitrary oriented in colour space is perturbatively considered.

Since the discovery of (anti-)instantons (the classical solutions of the Yang-Mills equations with nontrivial topological features) the problem of the gauge interaction between instantons is under intensive study and the dipole interaction of pseudo-particles with an external field has already been argued in the pioneering papers [1]. This practically important conclusion was grounded on using the would-be superposition ansatz for the approximate solution of the Yang-Mills equations

$$\mathcal{A}^a_\mu(x) = A^a_\mu(x) + B^a_\mu(x)\ . \tag{1}$$

Here the first term denotes the field of a single (anti-)instanton in the singular gauge as we are planning to consider an (anti-)instanton ensemble and need to have the non-trivial topology at the singularity point

$$A^a_\mu(x) = \frac{2}{g}\ \omega^{ab}\bar\eta_{b\mu\nu}\ \frac{\rho^2}{y^2+\rho^2}\ \frac{y_\nu}{y^2}\ , \tag{2}$$

where ρ is an arbitrary parameter characterizing the instanton size centered at the coordinate z and colour orientation defined by the matrix ω, $y = x - z$, $\bar\eta_{b\mu\nu}$ is the 't Hooft symbol (for anti-instanton $\bar\eta \to \eta$) and g is the coupling constant of non-abelian field. The second term describes an external field. For the sake of simplicity we consider, at first, external field originated by immovable Euclidean colour point-like source $e\delta^{3a}$ and Euclidean colour dipole $\pm e\delta^{3a}$ and limit ourselves dealing with $SU(2)$ group only. With this "set-up" we could claim now the results obtained before have maintained the interacting terms proportional e/g only and in this paper we are aimed to analyze the contributions proportional e^2 which display rather indicative behaviour of asymptotic energy of Euclidean non-abelian point-like sources while in the instanton liquid (IL) [2]. These Euclidean sources will generate the fields of the same nature as ones we face taking into account gluon field quantum fluctuations a_{qu} around the classical instanton solutions $A = A_{inst} + a_{qu}$.

In the Coulomb gauge the potentials take the following forms

$$B^a_\mu(x) = (\mathbf{0}, \delta^{a3}\ \varphi),\ \ \varphi = \frac{e}{4\pi}\ \frac{1}{|\boldsymbol{x}-\boldsymbol{z}_e|}\ ,$$

$$B^a_\mu(x) = (\mathbf{0}, \delta^{a3}\ \varphi),\ \ \varphi = \frac{e}{4\pi}\ \left(\frac{1}{|\boldsymbol{x}-\boldsymbol{z}_1|} - \frac{1}{|\boldsymbol{x}-\boldsymbol{z}_2|}\right)\ ,$$

for Euclidean point-like source and dipole in colourless state, respectively, with $\boldsymbol{z}_e$ being a coordinate of point-like source in peace and $\boldsymbol{z}_1$, $\boldsymbol{z}_2$ as coordinates characterizing a dipole. As known the field

originated by particle source in 4d-space develops the cone-like shape with the edge being situated just at the particle creation. Then getting away from that point the field becomes well established in the area neighboring to it and might be given by the Coulomb solution. In distant area where the field penetrates into the vacuum it should be described by the retarded solution. Here we are interested in studying the interactions in the neighboring field area and the cylinder-symmetrical field might be its relevant image in 4d-space. We treat the potentials in their Euclidean forms and then the following changes of the field and Euclidean point-like source variables are valid $B_0 \to iB_4$, $e \to -ie$ at transition from the Minkowski space. Actually, last variable change is resulted from the corresponding transformations of spinor fields $\psi \to \hat{\psi}$, $\bar{\psi} \to -i\hat{\psi}^\dagger$, $\gamma_0 \to \gamma_4$. It means we are in full accordance with electrodynamics where the practical way to have a pithy theory in Euclidean space is to make a transition to an imaginary charge.

Thus, the field strength tensor is given as

$$G^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g\ \varepsilon^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu\ , \tag{3}$$

where ε^{abc} is the entirely asymmetric tensor and for the field superposition of Eq.(1) it can be given by

$$\begin{aligned} G^a_{\mu\nu} &= G^a_{\mu\nu}(B) + G^a_{\mu\nu}(A) + G^a_{\mu\nu}(A,B)\ , \\ G^a_{\mu\nu}(A,B) &= g\ \varepsilon^{abc}(B^b_\mu A^c_\nu + A^b_\mu B^c_\nu)\ , \end{aligned} \tag{4}$$

if $G^a_{\mu\nu}(A)$ and $G^a_{\mu\nu}(B)$ are defined as in Eq.(3). Then gluon field strength tensor squared reads

$$\begin{aligned} G^a_{\mu\nu}\ G^a_{\mu\nu} &= G^a_{\mu\nu}(B)\ G^a_{\mu\nu}(B) + G^a_{\mu\nu}(A)\ G^a_{\mu\nu}(A) + G^a_{\mu\nu}(A,B)\ G^a_{\mu\nu}(A,B) + \\ &+ 2\ G^a_{\mu\nu}(B)\ G^a_{\mu\nu}(A) + 2\ G^a_{\mu\nu}(B)\ G^a_{\mu\nu}(A,B) + 2\ G^a_{\mu\nu}(A)\ G^a_{\mu\nu}(A,B)\ . \end{aligned} \tag{5}$$

The various terms of Eq.(5) provide the contributions of different kinds to the total action of full initial system of point-like sources and pseudo-particle

$$S = \int dx \left(\frac{1}{4}\ G^a_{\mu\nu}\ G^a_{\mu\nu} + j^a_\mu \mathcal{A}^a_\mu \right)\ . \tag{6}$$

So, the first and second terms of Eq.(5) provide the source self-energy (for the dipole it should be supplemented by the Coulomb potential of interacting sources) and the single instanton action $\beta = \frac{8\pi^2}{g^2}$. At small distances the first term is regularized by introducing the source "size" as in classical electrodynamics. These terms are proportional to e^2 and g^{-2}, respectively. The fourth and last terms of Eq.(5) together with the term $j^a_\mu A^a_\mu$ of Eq.(6) provide the contribution of proportional e/g and the third term of Eq.(5) only leads to the contribution proportional e^2 as the fifth term is equal zero owing to the gauge choice. Denoting the non-zero contributions as S_{int} we have after performing the calculations

$$S_{int} = \frac{e}{g}\ \bar{\eta}_{k4i}\ \omega^{3k} I_i + \left(\frac{e}{4\pi}\right)^2 J + \left(\frac{e}{4\pi}\right)^2 K_{kl}\ \omega^{3k}\omega^{3l}\ . \tag{7}$$

The exact form of I_i is unnecessary in this paper as applying the results for the IL model we have to average over the colour orientation of (anti-)instanton what leads to disappearance of dipole contribution. Two other terms are resulted from the "mixed" component of field strength tensor

$$\begin{aligned} G^a_{4i}(A,B) &= 2\ \frac{e}{4\pi}\ \varepsilon^{a3c}\ \omega^{ck}\ \bar{\eta}_{ki\alpha}\ \frac{y_\alpha}{y^2}\ \frac{\rho^2}{(y^2+\rho^2)}\ \frac{1}{|\boldsymbol{y}+\boldsymbol{\Delta}|}\ , \\ G^a_{4i}(A,B) &= 2\ \frac{e}{4\pi}\ \varepsilon^{a3c}\ \omega^{ck}\ \bar{\eta}_{ki\alpha}\ \frac{y_\alpha}{y^2}\ \frac{\rho^2}{(y^2+\rho^2)} \left(\frac{1}{|\boldsymbol{y}+\boldsymbol{\Delta}_1|} - \frac{1}{|\boldsymbol{y}+\boldsymbol{\Delta}_2|} \right)\ , \end{aligned} \tag{8}$$

$\boldsymbol{\Delta}_{1,2} = \boldsymbol{z} - \boldsymbol{z}_{1,2}$. The other contributions to $G^a_{\mu\nu}(A,B)$ are absent because of the gauge used. In order to handle the further formulae easily it is practical to introduce new dimensionless coordinates as $x/\rho \to x$. Then for the single source the function J and the tensor K take the following forms

$$J = 2 \int dy \; \frac{2\; y^2 - \boldsymbol{y}^2}{y^4 \; (y^2+1)^2 \; |\boldsymbol{y}+\boldsymbol{\Delta}|^2} \; ,$$

$$K_{kl} = 2 \int dy \; \frac{y_k y_l}{y^4} \; \frac{1}{(y^2+1)^2 \; |\boldsymbol{y}+\boldsymbol{\Delta}|^2} \; ,$$

and can not be integrated in the elementary functions. Fortunately, we need their asymptotic values at $\Delta \to \infty$ only

$$J \simeq \frac{5\pi^2}{2} \frac{1}{\Delta^2} \; ,$$

and for the components of the 2-nd rank tensor

$$K_{ij} = \delta_{ij}\; K_1 + \hat{\Delta}_i \hat{\Delta}_j \; K_2 \; ,$$

we have

$$K_1 \simeq \frac{\pi^2}{2} \frac{1}{\Delta^2} \; , \quad K_2 \simeq 0 \; .$$

Clearly, the mixed component of field strength tensor is of purely non-abelian origin but its contribution to the action of whole system (point-like sources and pseudo-particle) takes the form of self-interacting Euclidean source $\sim e^2$ although it is constructed by the instanton field and field generated by sources. Seems, this simple but still amazing fact was not explored properly.

Now we are trying to analyze the pseudo-particle behaviour in the field of Euclidean non-abelian source developing the perturbative description related to the pseudo-particle itself. Realizing such a program we "compel" the pseudo-particle parameters to be the functions of "external influence", i.e. putting $\rho \to R(x,z)$, $\omega \to \Omega(x,z)$. These new fields-parameters are calculated within the multipole expansion and then, for example, for the (anti-)instanton size we have

$$\begin{aligned} R_{in}(x,z) &= \rho + c_\mu \; y_\mu + c_{\mu\nu} \; y_\mu \; y_\nu + \dots \; , \qquad |y| \leq L \\ R_{out}(x,z) &= \rho + d_\mu \; \frac{y_\mu}{y^2} + d_{\mu\nu} \; \frac{y_\mu}{y^2} \frac{y_\nu}{y^2} + \dots \; , \qquad |y| > L \; , \end{aligned} \tag{9}$$

Similar expressions could be written down for the (anti-)instanton orientation in the colour space $\Omega(x,z)$ where L is a certain parameter fixing the radius of sphere where the multipole expansion growing with the distance increasing should be changed for the decreasing one being a result of deformation regularity constraint imposed. Then the coefficients of multipole expansion $c_\mu, c_{\mu\nu}, \dots$ and $d_\mu, d_{\mu\nu}, \dots$ are the functions of external influence. It turns out this approach allows us to trace the evolution of approximate solution for the deformed (crumpled) (anti-)instanton as a function of distance and, moreover, to suggest the selfconsistent description of pseudo-particle and point-like fields within the superposition ansatz of Eq.(1) (the paper has been completed recently).

Proceeding to the calculation of average energy of point-like source immersed into IL we have to remember that one should work with the characteristic configuration saturating the functional integral being the superposition of (anti-)instanton fields also supplemented by the source field $B^a_\mu(x)$, i.e.

$$\mathcal{A}^a_\mu(x) = B^a_\mu(x) + \sum_{i=1}^{N} A^a_\mu(x;\gamma_i) \; , \tag{10}$$

where $\gamma_i = (\rho_i, z_i, \omega_i)$ are the parameters describing the i-th (anti-)instanton. The IL density at large distances from source coincides practically with its asymptotic value $n(\Delta) \sim n_0 e^{\beta - S} \simeq n_0$ because

all action of any pseudo-particle there is approximately equal β. The quantity we are interested in should be defined by averaging S over the pseudo-particle positions and their colour orientations (taking all pseudo-particles of the same size) $\prod_{i=1}^{N} \frac{dz_i}{V}\, d\omega_i$, where V is the IL volume, and reads

$$E = \frac{\langle \int dx\; G^a_{\mu\nu} G^a_{\mu\nu} \rangle}{X_4} = \frac{e^2}{4\pi}\frac{1}{a} + \langle S' - \beta \rangle_3 \;.$$

The first term where a is a source "size" value (on strong interaction scale, of course) corresponds to the source energy and the second term presents contribution of all (anti-)instantons developing in thermodynamic limit the following form

$$\langle S' - \beta \rangle_3 \simeq \langle S_{int} \rangle_3 \sim n_3 \int d\boldsymbol{\Delta} \left(\frac{e}{4\pi}\right)^2 \left(J + \frac{K_{ii}}{3}\right) \;, \tag{11}$$

with $n_3 = n^{3/4}$ being the IL density in 3d space ($n = N/V$) and S' means the action without contribution of the first term of Eq.(5). In fact, the result can be reduced to the common denominator X_4 (playing the role of "time") as our concern here is (anti-)instanton behaviour in the source background when the field is well developed and the solution possesses an automodel property at any x_4 layer. The constant originated by the second term of Eq.(5) was omitted.

Then returning to the dimensional variables for the moment and using the asymptotic behaviours of corresponding magnitudes one can easily examine a linearly increasing contribution of source self-energy type

$$\langle S' - \beta \rangle_3 \sim \frac{6\pi^3}{\beta} \left(\frac{\bar\rho}{\bar R}\right)^3 \frac{1}{\bar\rho}\frac{L}{\bar\rho} = \sigma\; L \;,$$

where L is a formal upper limit of integration, $\bar\rho$ is the mean (anti-)instanton size , $\bar R$ denotes the pseudo-particles separation and it looks natural for the source to have $e = g$. For the $SU(N_c)$-group the denominator of Eq.(11) should be rearranged according to the change $3 \to N_c^2 - 1$ but there is no any noticeable impact on the result obtained. The "tension" coefficient at the parameters $\frac{\bar\rho}{\bar R} \simeq \frac{1}{3.67}$, $\beta \simeq 18$, $\bar\rho \simeq (1\ \text{GeV})^{-1}$ being characteristic for IL [3] takes the value of $\sigma \simeq 1$ GeV/fm what is in full accordance with the estimates extracted from the models for heavy quarkonia, for example. If one intends to explore the magnitudes like $\langle S_{int}\; e^{-S_{int}} \rangle$ (which could model an effect of suppressing pseudo-particle contribution in the source vicinity) in numerical calculations it becomes evident the linearly increasing behaviour starts to form at $\Delta/\bar\rho \sim 3$—4. The well known fact of area law absent for the Wilson loops in the pure gluodynamics could be explained by rapid decay ($\sim \Delta^{-4}$) of the corresponding correlators in IL and inadequate contribution of large size instantons. Apparently, for present consideration the result is rooted in non-abelian nature of gauge field and treating the approximate solution in the form of superposition ansatz of pseudo-particle and source fields. Perhaps, this picture could be made physically more transparent if other observables, for example, the non-abelian field flux over a surface are considered. However, we believe the result already obtained delivers one interesting message. The energy growth with the distance increasing teaches about impossibility of bringing the Euclidean colour source in IL as the source mass (the additional contribution received should be treated just in this way) is unboundedly increasing if the screening effects do not enter the play. This asymptotic estimate of Euclidean source energy in IL provides the major contribution to the generating functional in quasi-classical approximation if all the coupling constants are frozen at the scale of mean instanton size $\bar\rho$.

Now dealing with the field of colour dipole in the colour singlet state we are able to demonstrate the IL reaction once more. The contribution to average energy $\langle S' - \beta \rangle_3$ which we are interested in is defined at large distances by the integral very similar to that for the configuration with one single source. It has even the same coefficient but another averaging over the (anti-)instanton positions as

$$I_d = \int d\boldsymbol{\Delta}_1 \left(\frac{1}{\boldsymbol{\Delta}_1^2} - 2\, \frac{1}{|\boldsymbol{\Delta}_1||\boldsymbol{\Delta}_2|} + \frac{1}{\boldsymbol{\Delta}_2^2} \right) \;.$$

When the source separation $l = |\boldsymbol{z}_1 - \boldsymbol{z}_2|$ is going to zero, the field disappears and the final result should be zero. There are two parameters only to operate with the integral, they are L and l. The dimensional analysis teaches the integral is the linear function of both but the l-dependence only obeys the requirement of integral disappearance at $l \to 0$. It is easy to receive the equation

$$\frac{I_d}{4\pi} = L - 2\left(L - \frac{l}{2}\right) + L = l$$

for determining the coefficient (the contributions of three integrals are shown separately here). Finally the average dipole energy reads

$$\langle S' - \beta \rangle_3 \sim \sigma\; l\ .$$

Recently it becomes well known that for the SU(2)-group two point-like sources problem can be resolved for arbitrary orientation of the sources in colour space [4,5]. Moreover, the self-consistent scheme to describe the source (particle) and fields dynamics in the form of v/c-expansion can be elaborated as in electrodynamics [5]. Now in order to make the further conclusions understandable we remember some necessary results. Let us suppose in the points $\boldsymbol{z}_{1,2}$ the sources of intensity $e\widetilde{P}_1$, $e\widetilde{P}_2$ are situated, correspondingly, where the tilde sign on the top $\widetilde{P} = (P^1, P^2, P^3)$ means the vector in colour space with the unit normalization $|\widetilde{P}| = (P^\alpha P^\alpha)^{1/2} = 1$. These source vectors can be treated as a convenient basis to span the Yang-Mills solutions

$$\begin{aligned} \widetilde{B}_4 &= \varphi_1 \widetilde{P}_1 + \varphi_2 \widetilde{P}_2\ , \\ \widetilde{\boldsymbol{B}} &= \boldsymbol{a}\ \widetilde{P}_1 \times \widetilde{P}_2\ . \end{aligned} \tag{12}$$

If one requires now the fields are going to disappear at large distances from sources then the basis of vector-sources rotating around the constant vector $\widetilde{\Omega} = \overset{*}{\varphi}_1\ \widetilde{P}_1 + \overset{*}{\varphi}_2\ \widetilde{P}_2$ could correspond to such a choice of gauge

$$\begin{aligned} \dot{\widetilde{P}}_1 &= g\ \widetilde{B}_4(\boldsymbol{z}_1) \times \widetilde{P}_1\ , \\ \dot{\widetilde{P}}_2 &= g\ \widetilde{B}_4(\boldsymbol{z}_2) \times \widetilde{P}_2\ , \end{aligned} \tag{13}$$

with the frequency $g|\widetilde{\Omega}|$ where $\overset{*}{\varphi}_1 = \varphi_1(\boldsymbol{z}_2)$, $\overset{*}{\varphi}_2 = \varphi_2(\boldsymbol{z}_1)$. The vectors dotted on their top mean the differentiation in x_4. The same character of solutions persist at transition to the Minkowski space. The functions $\varphi_{1,2}$ and vector-function $\boldsymbol{a}$ are dependent on $\boldsymbol{x}$ only and are determined by the following equations

$$\begin{aligned} \boldsymbol{D}\boldsymbol{D}(\varphi - \overset{*}{\varphi}) &= -e\delta \\ \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{a} &= \boldsymbol{j}, \end{aligned} \tag{14}$$

where $\boldsymbol{D}_{kl} = \nabla\delta_{kl} + g\boldsymbol{a}C_{kl},\ k, l = 1, 2$ and the current is defined as

$$\boldsymbol{j} = g\ (\varphi - \overset{*}{\varphi}) J \boldsymbol{D}(\varphi - \overset{*}{\varphi})\ .$$

Besides, the operator $\boldsymbol{D}$ is also acting on

$$\varphi^{\mathrm{T}} = \|\varphi_1, \varphi_2\|\ ,\quad \overset{*}{\varphi}{}^{\mathrm{T}} = \|\ \overset{*}{\varphi}_1, \overset{*}{\varphi}_2\ \|\ ,\quad \delta^{\mathrm{T}} = \|\delta(\boldsymbol{x} - \boldsymbol{z}_1), \delta(\boldsymbol{x} - \boldsymbol{z}_2)\|\ .$$

The matrices C and J are defined as

$$C = \left\| \begin{array}{cc} -(\widetilde{P}_1\widetilde{P}_2) & -(\widetilde{P}_2\widetilde{P}_2) \\ (\widetilde{P}_1\widetilde{P}_1) & (\widetilde{P}_1\widetilde{P}_2) \end{array} \right\|\ , \qquad\qquad J = \left\| \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right\|\ .$$

It is resulted from Eq.(13) the modules of vectors $P_{1,2}$ and their scalar product (P_1P_2) do not change in "time". The system of Eq.(14) has pretty transparent physical meaning. The colour field originated by two point-like sources is an origin of colour charge itself because the gluons are not neutral. The self-consistent picture of charges and corresponding currents is set up in between the initial charges. The solutions of this system were accurately investigated both analytically and numerically and it has been found out the sources are interacting in the Coulomb-like way at any magnitude of coupling constant g and when the coupling constant is not large $\frac{g}{4\pi} < \sqrt{2}$, the solutions are well approximated by simple Coulomb-like potentials

$$\varphi_{1,2} = \frac{e}{4\pi}\ \frac{1}{|\boldsymbol{x} - \boldsymbol{z}_{1,2}|}$$

with $\boldsymbol{a}$ defined by the current density generated by these potentials. In general, the vector field looks like the field of constant magnet with the poles placed in the points $\boldsymbol{z}_1, \boldsymbol{z}_2$. Then at the straight line connecting sources the field develops the longitudinal component of constant value only

$$|\boldsymbol{a}_{\|}| = \frac{e}{4\pi}\ \frac{eg}{|\boldsymbol{z}_1 - \boldsymbol{z}_2|}\ .$$

Going away from this straight line the field is rapidly getting weaker. The total energy stored in the colour field is evaluated as

$$E_{cf} = \int d\boldsymbol{x}\ \frac{\widetilde{\boldsymbol{E}}^2 + \widetilde{\boldsymbol{H}}^2}{2} \sim e^2\ \frac{(\widetilde{P}_1\widetilde{P}_2)}{l} + g^2e^4\ I\ \frac{(\widetilde{P}_1\times\widetilde{P}_2)^2}{l}$$

where $I = \pi\ (6-\pi^2/2)$. Apparently, we did not include the terms corresponding to the self-interaction of sources. When the source separation becomes large enough the frequency of precession is swiftly going to zero $\sim 1/l$, and the basis composed by the vectors $\widetilde{P}_1, \widetilde{P}_2, \widetilde{P}_1\times\widetilde{P}_2$ could be approximated as being in peace. The contribution of nonsingular vector field (comparing to the Coulomb field, see Eq.(8)) to the interaction with (anti-)instanton may be neglected. Then the simple superposition

$$\widetilde{B}_4 = \varphi_1\widetilde{P}_1 + \varphi_2\widetilde{P}_2$$

might be used as an approximate solution. Clearly, the contribution of such a dipole to the mean energy of IL at large distances is proportional to

$$\frac{I_d}{4\pi} = (\widetilde{P}_1\widetilde{P}_1)L + 2\ (\widetilde{P}_1\widetilde{P}_2)\left(L - \frac{l}{2}\right) + (\widetilde{P}_2\widetilde{P}_2)L = (\widetilde{P}_1 + \widetilde{P}_2)^2\ L - (\widetilde{P}_1\widetilde{P}_2)\ l\ .$$

At small distances where the Coulomb-like fields are large the (anti-)instantons are strongly suppressed and this factor together with the contribution of vector field $\boldsymbol{a}$ should be taken into account. Here we limit ourselves with this estimate obtained only. Surely, the result demonstrates again that it is very difficult to reveal the states with open colour in IL and there is the small parameter in the problem. Indeed, the deviations of vector-sources from the anti-parallel orientations may not be large since it is reasonable to estimate $L \sim R_D$ screening radius for which in the approach developed $R_D \geq m_p/\sigma$ is valid where m_p is the mass of lightest stable (on the scale of strong interaction) particle of non-Goldstone nature.

Thus, we described all field states at any positions of sources $\boldsymbol{z}_1$, $\boldsymbol{z}_2$ for any source orientations $\widetilde{P}_1$, $\widetilde{P}_2$. The estimate of these configurations contribution to the generating functional can be performed within the variational maximum principle. Then the calculated mean energy of Euclidean sources appears in the exponential factor and with a precision up to several inessential terms may be presented as the suppression factor for the states with open colour

$$Z \geq e^{-EX_4}\ ,$$

we omitted the condensate contribution and the part of fermion component here. The sources could be interpreted as non-relativistic particles if they have large masses $m_{1,2}$ and their coordinates are the functions of "time" x_4 similar to their states in colour space where they are described by the following spinors $u^T = (1,0)$ and $\bar{u}^T = (0,1)$. Assuming the changes of these states to be insignificant we describe their supposed evolution perturbatively with the matrices $U \simeq 1 + i\ \boldsymbol{\sigma}\boldsymbol{\lambda}/2$, $V \simeq 1 + i\ \boldsymbol{\sigma}\boldsymbol{\mu}/2$ (here $\boldsymbol{\sigma}$— are the Pauli matrices) for the first and second particles, respectively. As the result we have for the vector-source of first particle $P_1^a \simeq \delta^{3a} - \varepsilon^{3ab}\lambda^b$ and similarly for the second one $P_2^a \simeq -\delta^{3a} - \varepsilon^{3ab}\mu^b$.

Hence, if the suppression factor is interpreted according to the potential model philosophy then the generating functional in the form

$$
\begin{aligned}
&Z = \int D[\boldsymbol{\lambda}]\ D[\boldsymbol{z}_1]\ D[\boldsymbol{\mu}]\ D[\boldsymbol{z}_2]\ e^{-S}\ , \\
&S \simeq \int dx_4 \left(\frac{\boldsymbol{p}_1^2}{2m_1} + \frac{\boldsymbol{p}_2^2}{2m_2} + \sigma\ (\widetilde{P}_1 + \widetilde{P}_2)^2\ R_D - \sigma\ (\widetilde{P}_1\widetilde{P}_2)\ |\boldsymbol{z}_1 - \boldsymbol{z}_2| + e^2 \frac{(\widetilde{P}_1\widetilde{P}_2)}{|\boldsymbol{z}_1 - \boldsymbol{z}_2|} \right)\ ,
\end{aligned}
\tag{15}
$$

will be adequate to the quantum mechanical system of coloured particles. It describes the evolution of the states $\Psi(x_4; \boldsymbol{z}_1, \boldsymbol{z}_2; \boldsymbol{\lambda}, \boldsymbol{\mu})$ where $\boldsymbol{p}_i$ are the particle momenta (the particle spins are not taken into account). Finally, the generating functional should be presented as an integral over "coloured" spinors. However, Eq.(15), seems, underestimates the factor of "coloured" spinor evolution because the first and second components of the vectors $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ only are essential at integrating, i.e. Eq.(15) should be taken as an approximate expression. The Coulomb-like part of this expression looks similar to the electrodynamics and in the limit of the IL density going to zero the expression should properly reproduce the results for the (anti-)parallel sources.

Nevertheless, very instructive message of Eq.(15) is that the system prefers to evolve over colourless states. The probability of getting the source orientation out of the anti-parallel position is strongly suppressed by the factor σR_D. It is clear the problem admits various generalizations (number of particles, identical particles, etc) but apparently the result will provide again a strong hint that the integrations over the colourless states only are essential. In a sense, it demonstrates a sort of duality of integrating over the "coloured" spinors and the colourless (hadronic) states. Concluding we would like to emphasize the proposed approximation for calculating the generating functional for sources of Euclidean non-abelian field within quasi-classical approach leads to rather indicative picture of the dynamics of the objects resembling many features of strongly interacting particle phenomenology.

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